Domain and Range of Quadratic Functions

Recall the definitions of domain and range of a relation:
- the domain of a relation is the set of values of the independent variable for which the relation is defined
- the range of a relation is the set of dependent variable values in the relation; note that the range will depend on the defining equation of the relation

For the function \( f(x) = a(x-h)^2 + k \), the domain is all the real numbers (since the graph of this function, which is a parabola, extends forever horizontally in both directions). The range, however, includes
- only the real numbers which are greater than or equal to \( k \) (the \( y \)-coordinate of the vertex), if the parabola opens upward; or
- only the real numbers which are less than or equal to \( k \) (if the parabola opens downward).

For an example relating the domain and range to the graph of a quadratic function, see Example 2 on page 61 of your textbook.

In many real-world situations, a quadratic function is a good model of what is happening, but the domain and range are restricted by the nature of the problem being considered.

Consider the case of an object being shot into the air, where we are interested in its height above the ground, \( h \), as a function of the time since it was launched, \( t \). The domain of this problem will be all the time values from launch (when \( t = 0 \)) up to the time when it lands. We do not consider negative values of time, nor are we interested in time values after the object has hit the ground.

The graph describing the path of the object is a parabola that opens downward. Normally, we say the range of the corresponding function is all real values less than or equal to the \( y \)-coordinate of the vertex. But since the domain is restricted for this problem, the range will also be restricted.